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ASSAULT BOAT EQUATIONS

Robert W. Strother

Naval Training Equipment Center
Orlando, Florida

November 1972

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Technical Report: NAVTRAEQUIPCEN IH-203

**ASSAULT BOAT EQUATIONS
INTERIM REPORT**

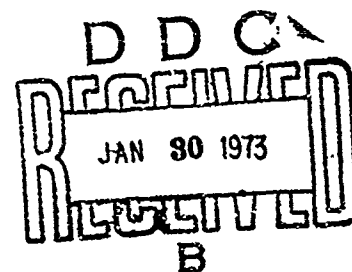
Robert W. Strother

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Naval Training Equipment Center
Orlando, Florida 32813
NAVTRAEQUIPCEN TASK NO. 1747-04**

November 1972

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13. ABSTRACT A description is given on the analysis and modification to existing equations of motion for a typical assault boat vehicle. Furthermore, a brief description is given of a real-time, assault boat simulation program that will drive a mockup of an assault boat trainer at the Naval Training Equipment Center. This is part of a research effort to satisfy the Navy's requirement for an assault boat trainer.			

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Technical Report: NAVTRAEQUIPCEN IH-203

ASSAULT BOAT EQUATIONS
(Interim Report)

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NAVTRAEQUIPCEN IH-203

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SECTION I

INTRODUCTION

This is an interim report on the Assault Boat Equations, Task 1747-04. Work is now going on within the Naval Training Equipment Center (NAVTRAEQUIPCEN) to satisfy the Navy's requirement for an assault boat coxswain trainer. An assault boat breadboard device has been installed and is to be driven by the Computer Laboratory Sigma 7 computer system. The present task entails development and implementation of a requisite real-time computer program based on landing and retracting behavior of an assault boat from a beach. Initially, the assault boat equations and experimental data were provided to the NAVTRAEQUIPCEN by references 1 and 2. The experimental data, which were obtained by Oceanics, Incorporated from the British Hovercraft Corporation, Limited, of England, were gathered from tank tests employing a scale model of a typical assault boat vehicle.

The Computer Laboratory's role in the project is twofold: first is the modification, where needed, and verification of the assault boat mathematical model; and second the development of a real-time simulation program to drive the assault boat breadboard simulator being built at the NAVTRAEQUIPCEN.

This report describes the in-house work with the exception of detailed software descriptions performed by the author under the direction of the project engineer, H. C. Saltzman. Detailed software descriptions will appear in internal documentation.

SECTION II

STATEMENT OF THE PROBLEM

The problem was twofold. First, a real-time simulation program, based on the assault boat mathematical model, had to be written to run on the Sigma 7 computer. Continuous and real-time control (i.e., instantaneous input/output) of the assault boat mathematical model was required during verification tests of the equations of motion.

The second problem was the verification of the equations of motion for the assault boat. However, there were many inconsistencies found between the assault boat mathematical model and the experimental data. As a result, part of the problem was to make modifications to the assault boat equations of motion and then to perform verification tests based on available experimental data.

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SECTION III

METHOD OF PROCEDURE

The procedure was initially to write a nonreal-time Fortran program that ran on the Sigma 7 computer. The purpose of this was to make an off-line study of the equations of motion. Next, a real-time simulation program with on-line control was written. Approximately one-third of the program was written in Fortran IV and two-thirds in assembly language. With no further addition, the program has a cycle time of 60msec. The input controls, throttle and rudder angle, were simulated by the Sigma 7 computer rather than inputted by the assault boat hardware. For off-line study, the card reader and line printer can be used as input/output devices. A time history of inputs can be read into the simulation program by the card reader. Furthermore, the simulation program (if called upon by the operator) will output up to eight variables of motion to an analog strip recorder. The executive program of the simulation program interfaces with the external sense switches on the console to allow the operator to select the input/output modes described above.

After the simulation program was operating correctly, further tests on the validity of the equations of motion were carried out. Because of the discrepancies found between the experimental variables and the theoretical numerical values of motion variables, modifications to the coefficients were made, and in some cases new equations were derived based on curve fitting equations to the experimental data.

SECTION IV

RESULTS

A. SUMMARY OF EQUATIONS.

At the time of this report, the following is a summary of the assault boat equations used. The glossary contains a definition of the variables.

Accelerations

$$\dot{U} = \frac{1}{m-x_u} (X_{hull} + X_{props} + X_{rudder} + X_{wind} + X_{waves} + X_{beach} + mvr) \quad (1)$$

$$\dot{V} = \frac{1}{m-y_v} (Y_{hull} + Y_{props} + Y_{rudder} + Y_{wind} + Y_{waves} - mur) \quad (2)$$

$$\ddot{Z} = \frac{1}{m+2A'_{33}} (-2A'_{33}\dot{\Theta} - 2N'_{zz}(\dot{Z}+U\Theta) - 2pg(Bz+Z_{waves})) \quad (3)$$

$$\ddot{\phi} = \frac{1}{I_{xx}-K_p} (K_p\dot{\phi} - W|GM|\phi + K_{waves}) \quad (4)$$

$$\ddot{\Theta} = \frac{1}{I_{yy}+2A'_{33}} (2A'_{33}(U\dot{Z}+U^2\Theta) - \frac{2}{3}N'_{zz} - \frac{2}{3}gB\dot{\Theta} + M_{waves}) \quad (5)$$

$$\dot{r} = \frac{1}{I_{zz}} (N_{hull} + N_{props} + N_{rudder} + N_{control} + N_{wind} + N_{waves}) \quad (6)$$

Hull Forces

$$X_{hull} = X_1u|v| + X_2vr \quad (\text{Ahead}) \quad (7)$$

$$X_{hull} = X_2vr + X_3v^2 \quad (\text{Astern}) \quad (8)$$

$$Y_{hull} = Y_v\dot{V} + Y_1uv + Y_2ur \quad (\text{Ahead, Astern}) \quad (9)$$

$$N_{hull} = N_r\dot{r} + N_1ur + N_2r|r| + N_3uv \quad (\text{Ahead, (Astern)}) \quad (10)$$

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Propeller Forces

$$X_{\text{props}}^{(1)} = 10.5n_o - .2n_o u^2 - u^3 \quad (\text{ahead, deep water}) \quad (11)$$

$$X_{\text{props}}^{(2)} = X_{\text{props}}^{(1)} + .08u n_o^2 - 77.37u^2 n_o \left(\frac{\alpha}{h} \right) \quad (\text{ahead, shallow water}) \quad (12)$$

$$Y_{\text{props}} = .0073u^4 + 0.20u^2 \quad (\text{ahead}) \quad (13)$$

$$N_{\text{props}} = 0.1818u^4 + 5.308u^2 \quad (\text{ahead}) \quad (14)$$

$$X_{\text{props}}^{(1)} = 9n_o^2 - n_o u^{1.5} - 7.1022u^2 \quad (\text{astern, deep water}) \quad (15)$$

$$X_{\text{props}}^{(2)} = X_{\text{props}}^{(1)} + .11n_o^2 u - 62.86u^2 n_o \left(\frac{\alpha}{h} \right) \quad (\text{astern, shallow water}) \quad (16)$$

$$Y_{\text{props}} = 6.91u + 0.58u^2 \quad (\text{astern}) \quad (17)$$

$$N_{\text{props}} = 156.91u + 15.78u^2 \quad (\text{astern}) \quad (18)$$

$$n_o = \text{minimum}(n_p, n_s) + 0.8|n_s - n_p| \quad (\text{astern, ahead}) \quad (19)$$

Rudder Forces

$$X_{\text{rudders}} = -2.8u^2 \quad (\text{ahead, astern}) \quad (20)$$

$$Y_{\text{rudders}} = 2.71u^3 \quad (\text{ahead}) \quad (21)$$

$$Y_{\text{rudders}} = 0.47u^2 \quad (\text{astern}) \quad (22)$$

$$N_{\text{rudders}} = -56.69u^3 \quad (\text{ahead}) \quad (23)$$

$$N_{\text{rudders}} = 12.28u^3 \quad (\text{astern}) \quad (24)$$

Control Forces

$$N_{\text{control}} = 1.25 (T(n_p) - T(n_s)) \quad (\text{ahead}) \quad (25)$$

$$N_{\text{control}} = 1.25 (T(n_s) - T(n_p)) \quad (\text{astern}) \quad (26)$$

$$T(n) = 9.95n_o^2 \quad (\text{ahead}) \quad (27)$$

$$T(n) = .95n_o^3 \quad (28)$$

Wind Forces

$$X_{\text{wind}} = -\frac{\rho_a}{2} C_D (A_1 |\cos \mu| + A_2 |\sin \mu|) U_{\text{RW}}^2 \cos \mu \quad (29)$$

$$Y_{\text{wind}} = \frac{\rho_a}{2} C_D (A_1 |\cos \mu| + A_2 |\sin \mu|) U_{\text{RW}}^2 \sin \mu \quad (30)$$

$$N_{\text{wind}} = 0.3L \frac{\rho_a}{2} C_D (A_1 |\cos \mu| + A_2 |\sin \mu|) U_{\text{RW}}^2 \cdot (1 - |\sin \mu|) \text{sign}(\sin \mu) \quad (31)$$

$$U_{\text{RW}} = \sqrt{U_{\text{RW}}^2 + v_{\text{RW}}^2} \quad (32)$$

$$u_{\text{RW}} = U_{\text{W}} \cos(\psi_{\text{W}} - \psi) - u \quad (33)$$

$$v_{\text{RW}} = U_{\text{W}} \sin(\psi_{\text{W}} - \psi) - v \quad (34)$$

$$= \tan^{-1} \left\{ \frac{-v_{\text{RW}}}{u_{\text{RW}}} \right\} \quad (35)$$

Kinematics (position of boat relative to the inertia reference frame at time t_n)

$$X_o(t_n) = X_o(t_{n-1}) + u \Delta t \cos \psi - v \Delta t \sin \psi \quad (36)$$

$$Y_o(t_n) = Y_o(t_{n-1}) + v \Delta t \cos \psi + u \Delta t \sin \psi \quad (37)$$

$$\Delta t = t_n - t_{n-1} \quad (38)$$

B. MODIFICATIONS TO THE EQUATIONS OF MOTION AND TEST RESULTS.

The experimental data used as a test criteria were obtained by Oceanics, Incorporated from the British Hovercraft Corporation, Limited. This experimental data was obtained from a scale model of landing craft model (LCM) during tank tests and the results can be found in reference 1.

Free oscillation tests were performed to obtain the natural frequencies for the simulation and to compare these results with the experimental values.

MOTION	SIMULATION	EXPERIMENTAL
Heave	2.8sec	2.9sec
Roll	3.3sec	3.1sec
Pitch	2.1sec	3.4sec

Figure 1 shows a graph of these displacements as a function of time. Figures 2 through 5 show the variation of RPM with speed at self propulsion for various conditions. These tests were carried out by setting the RPM of the assault boat propellers to a fixed value and then recording the constant velocity obtained while maintaining a straight course with the rudders. When these tests first were run, the equations as provided by Oceanics, Incorporated were erroneous. However, this situation was rectified by deriving two new equations for the propeller force in the X-direction of the body axis to replace the two equations obtained from Oceanics, Incorporated.

$$X_{\text{props}} = 4.93n_0^2 + .82u^{3/2}n_0 - .06u^3 \quad (\text{Ahead, deep water})$$

was replaced by

$$X_{\text{props}} = 10.5n_0^2 - .2u^2n_0 - u^3 \quad (\text{Ahead, deep water})$$

Also,

$$X_{\text{props}} = 16.36n_0^2 - 17.97un_0 - 3.38u^2 \quad (\text{Astern, deep water})$$

was replaced by

$$X_{\text{props}} = 8.0n_0^2 - .9u^{1.5}n_0 - 7.1022u^2 \quad (\text{Astern, deep water})$$

Two equations for the propeller force are required because the basic framework of the mathematical model treats the approach to and retraction from the beach as two separate simulation problems. One set of equations is used when the assault boat approaches the beach and the other when the assault boat retracts from the beach. This presents no problem. However, Oceanics, Incorporated treated propeller force in the body X-direction as if both the port and starboard propellers were rotating at the same RPM. As an outcome, their expression for combined propeller rotation, $n_0 = \frac{1}{2}(n_p + n_s)$, where n_p and n_s equal port and starboard rotation respectively, was found to be inappropriate under the following condition. That is, when considering the net X-body propeller force due to a differential in the RPM of the propellers, for example, just the starboard propeller rotating, error in the forward velocity of the boat was introduced. However, another expression for propeller rotation was derived $n_0 = \text{MINIMUM}(n_p, n_s) + .8|n_p - n_s|$.

This expression gave excellent results as can be seen from figures 2 through 5. It will be noted that when the propellers are rotating in unison, i.e., when $n_p = n_s$, the equations for n_0 reduces to $\frac{1}{2}(n_p + n_s)$ which is the original expression provided by Oceanics, Incorporated. Furthermore, the new expression

for propeller rotation (when used in the propeller force equation) gave excellent results for the variation of RPM with speed when just the starboard screw was rotating. This can be seen in figures 3 and 5.

Figures 7 and 8 show resistance as a function of velocity for both ahead and astern motion. The original expressions of resistance to motion for both ahead and astern motion were inadequate. The inadequacy of the drag in the water became apparent in deceleration when both propellers were turned off. However, when the equations of resistance were altered to fit the resistance data as a function of forward velocity, deceleration became more pronounced.

Figure 10 shows the variation of side force with speed. However, in order to obtain this "good fit", it was necessary to derive a new equation for the propeller force in the Y-body axis system.

The propeller force in the Y-body axis system,

$$Y_{\text{props}} = 7.13u + 5.74u^2 - .75u^3 + .03u^2,$$

was replaced by

$$Y_{\text{props}} = .0073u^4 + .20u^2.$$

Figure 11 shows the variation of yawing moment with speed for ahead motion and both propellers rotating in calm water. As can be seen, there is close agreement between theoretical and experimental values. However, as before, it was necessary to derive a new equations for yawing moment produced by the propellers. The yawing moment about the Z-body axis system produced by the propellers,

$$N_{\text{props}} = 103.13u - 114.2u^2 + 15.61u^3 - .75u^4,$$

was replaced by

$$N_{\text{props}} = .1818u^4 + 5.308u^2.$$

Figure 12 shows the variation of side force and yawing moment with speed for astern motion and both propellers rotating in calm water. The original equations provided by Oceanics, Incorporated were not modified since there was close agreement between theoretical and experimental values. The dispersion in experimental data was due to aeration effects during tank tests on the landing craft model.

Figure 13 shows the difference between the port and starboard RPM as a function of speed when a straight course is maintained. However, these comparisons of theoretical versus experimental values were made off-line as follows. Since a straight course is maintained ahead, $\dot{r} = r = \dot{v} = 0$ in the coupled equations of lateral motion (sway) and yaw with a constant forward velocity, u , and lateral velocity, v , maintained. With the above conditions, the equations of motion reduce to $u = \text{constant}$ (i.e., constant forward speed and no yawing)

$$Y_{\text{uv}} + Y_{\text{props}} = 0 \text{ (i.e., zero lateral acceleration and no yawing)}$$

$$N_{3uv} + N_{\text{props}} + N_{\text{controls}} = 0 \text{ (i.e., zero yaw rate)}$$

where

$$N_{\text{controls}} = 1.25(T(n_p) - T(n_s)),$$

$$T(n) = 9.95n^2, \text{ and } n_p, n_s = \text{port and starboard propeller RPS.}$$

By inputting an initial forward velocity, u , and starboard propeller, n_s , an equilibrium lateral velocity, v , and starboard port propeller, n_p , can be calculated. This off-line method was used to calculate the theoretical values for the difference of port and starboard propeller as a function of forward speed when a straight course was maintained.

Figure 14 shows the variation of side force with speed and rudder angle. The solid lines present the experimental data from the simulation mathematical model. No modifications were made to the equation that provides the side force produced by the rudder.

Figure 15 shows the variation of yawing moment with speed and rudder angle. Again, no modifications to the equation that calculates the rudder yawing moment was made.

SECTION V

CONCLUSIONS

In conclusion, a preliminary simulation assault boat model has been completed. There is an abundance of experimental data available in reference 2 on the performance of an assault boat model in calm water, regular waves, and the landing and retraction of the boat from the beach. Hence, there is a good foundation for improving and testing the validity of further changes to the assault boat vehicle equations of motion in order to improve the simulation. One major shortcoming exists in the experimental data; there are no acceleration and deceleration curves on the assault boat model. A major shortcoming of the math model is that although differences in port and starboard propellor rotation are permitted, differences in the direction of propeller rotation is not permitted. This is a serious flaw in that the propellers are the main control the coxswain has over the control of the assault boat. This deficiency will be corrected.

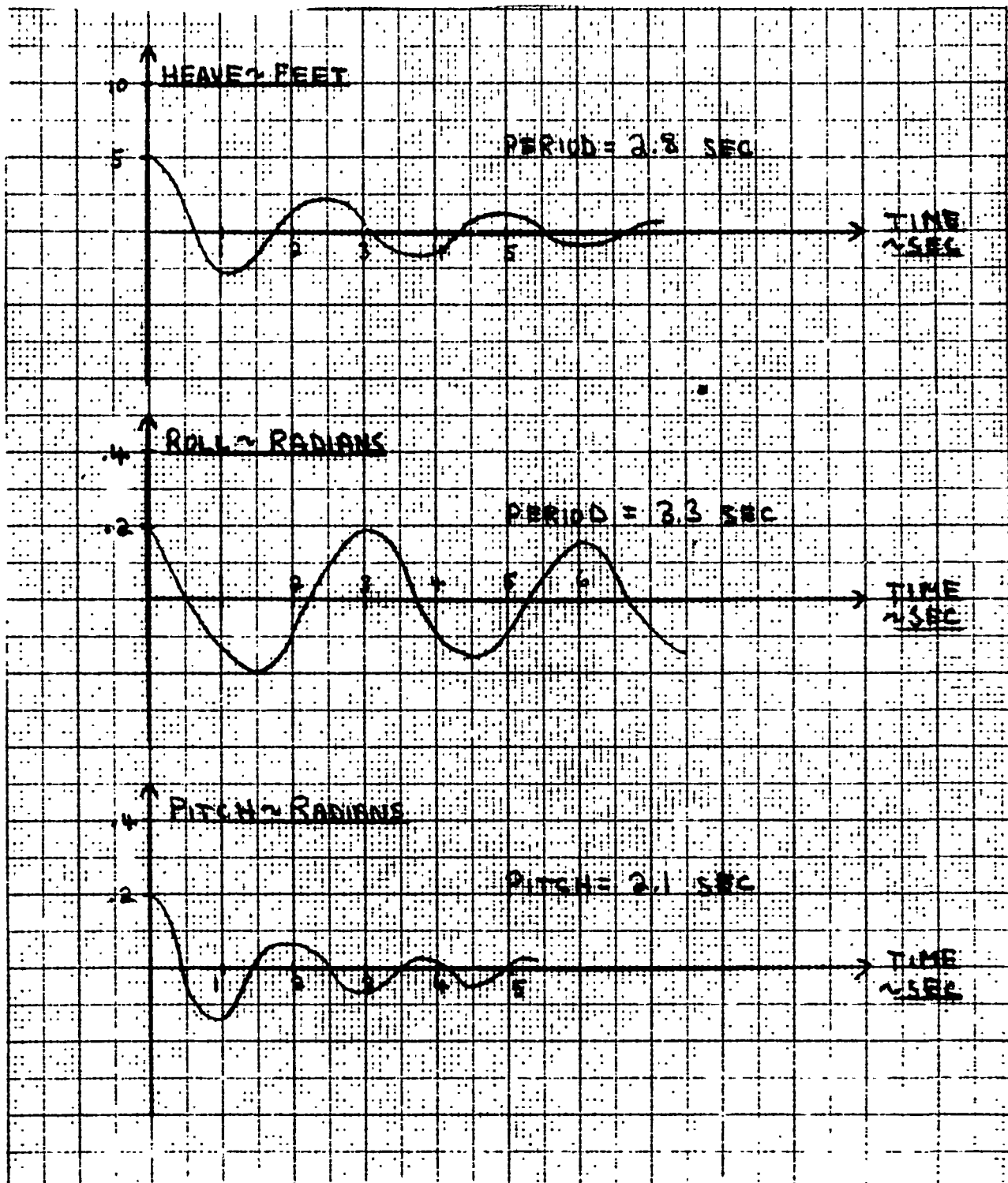


Figure 1. Computer Simulation of LCM Free Oscillation Calm Water

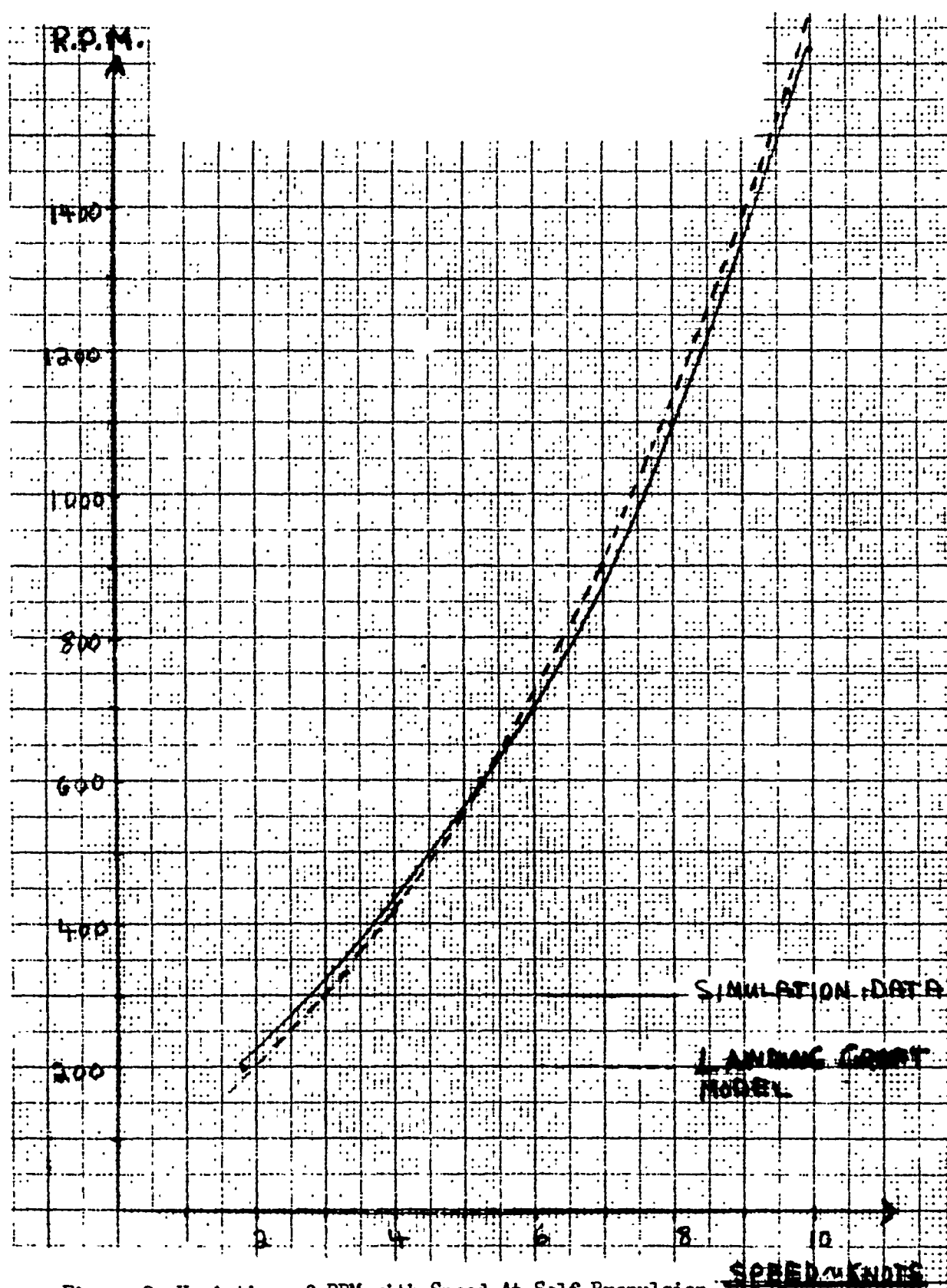


Figure 2. Variation of RPM with Speed At Self Propulsion
Ahead Twin Screws Calm Water

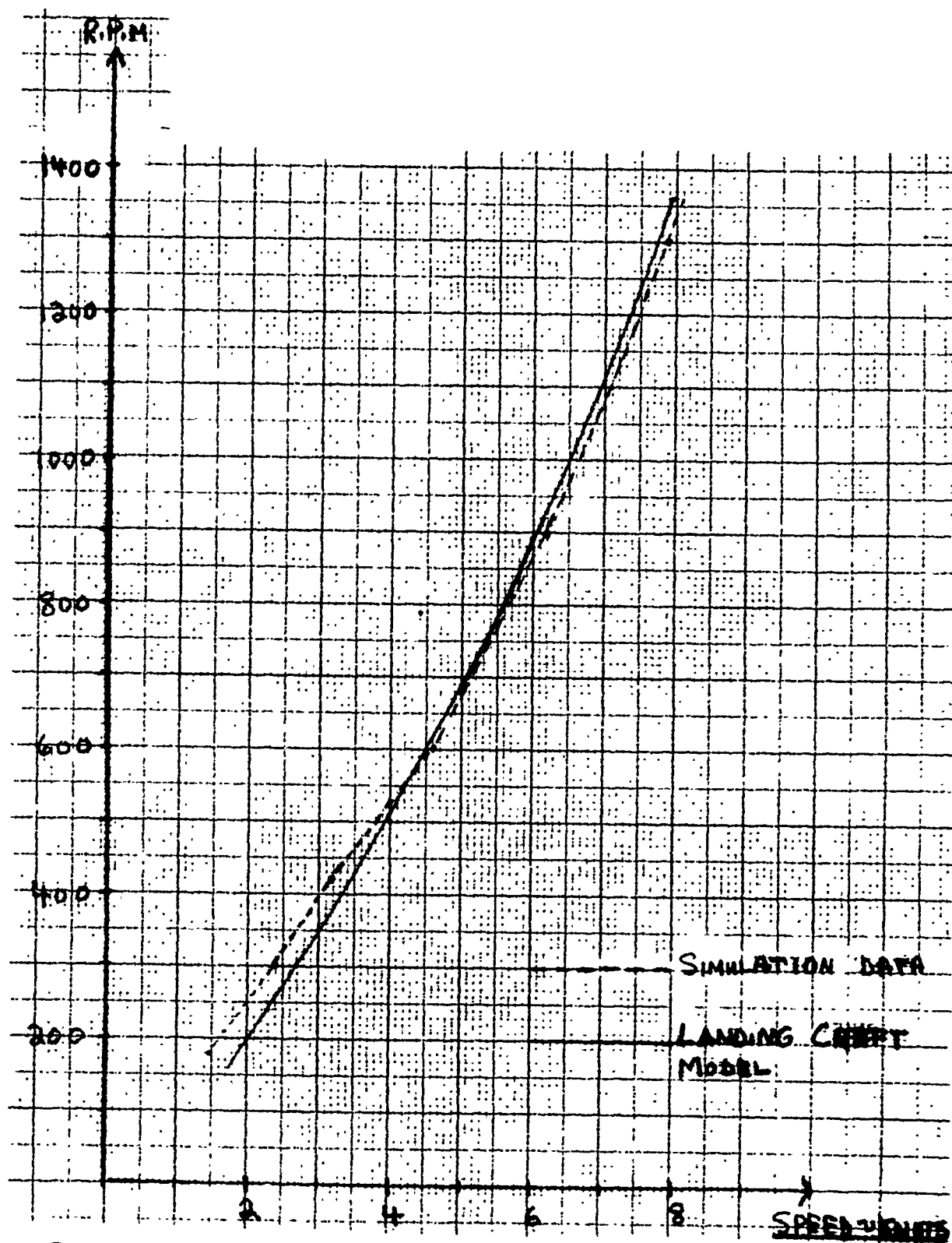


Figure 3. Variation of RPM with Speed at Self Propulsion Ahead
Starboard Screw Calm Water

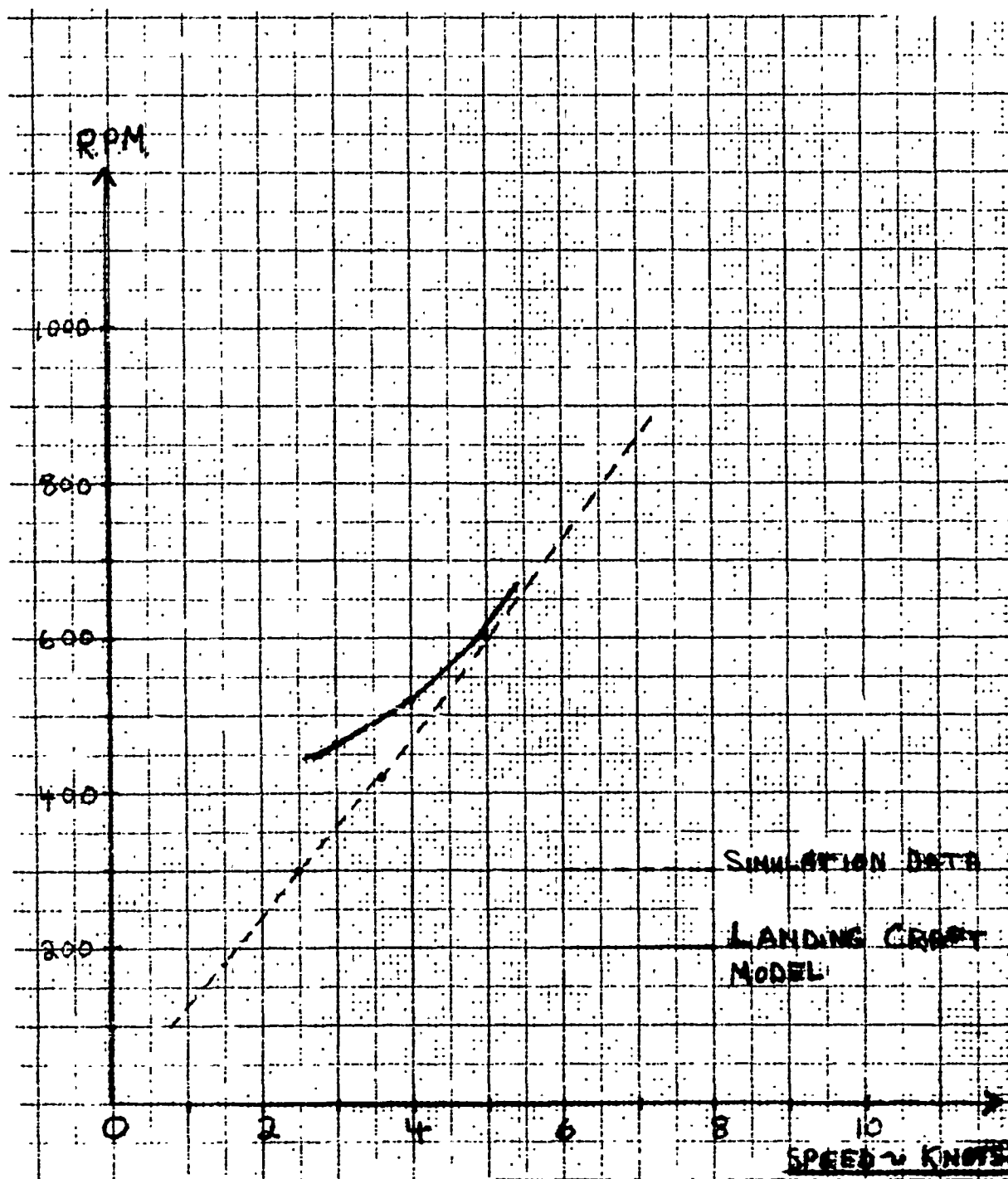


Figure 4. Variation of RPM with Speed at Self Propulsion
Calm Water Astern Twin Screws

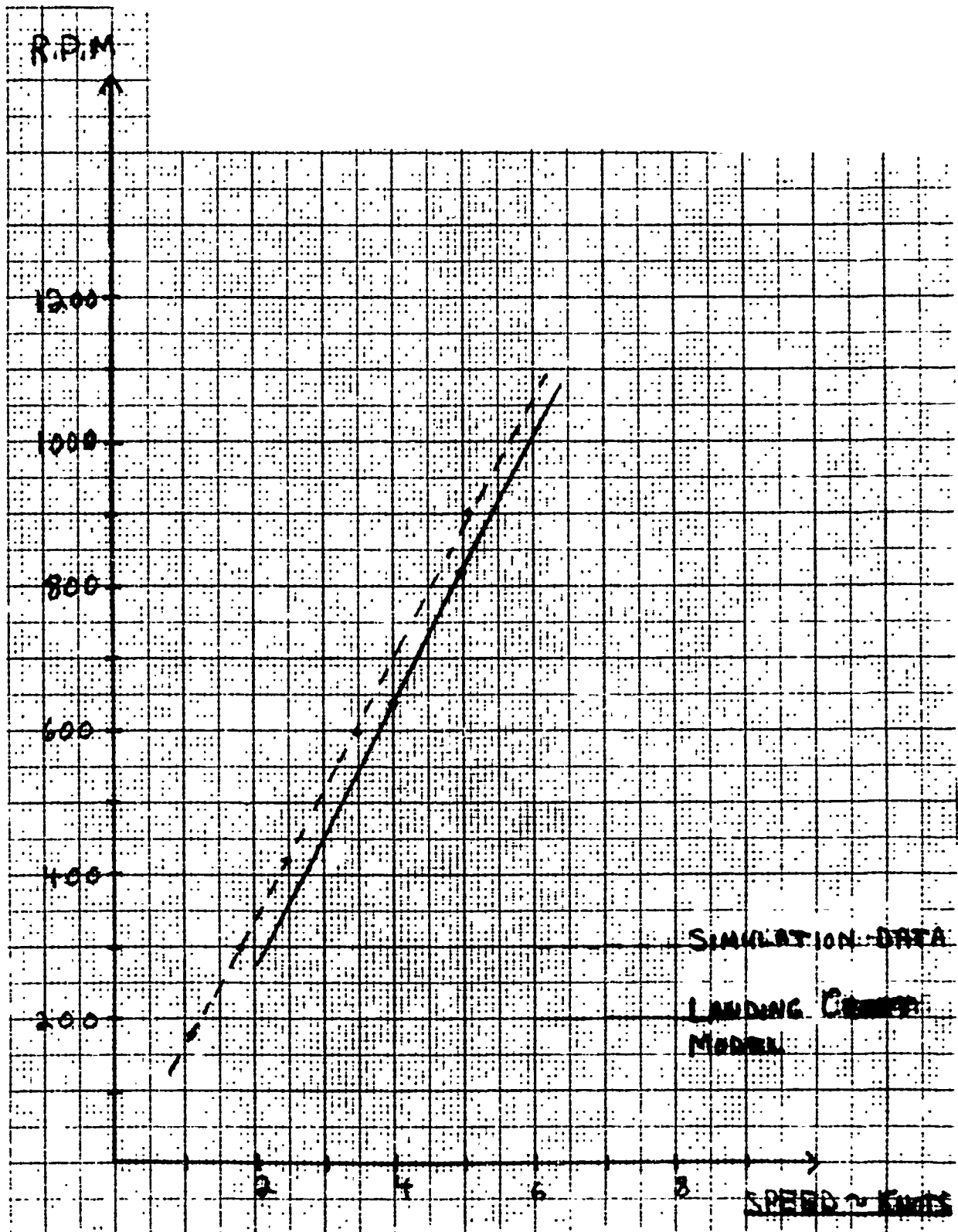


Figure 5. Variation of RPM with Speed at Self Propulsion Astern Starboard Screw Calm Water

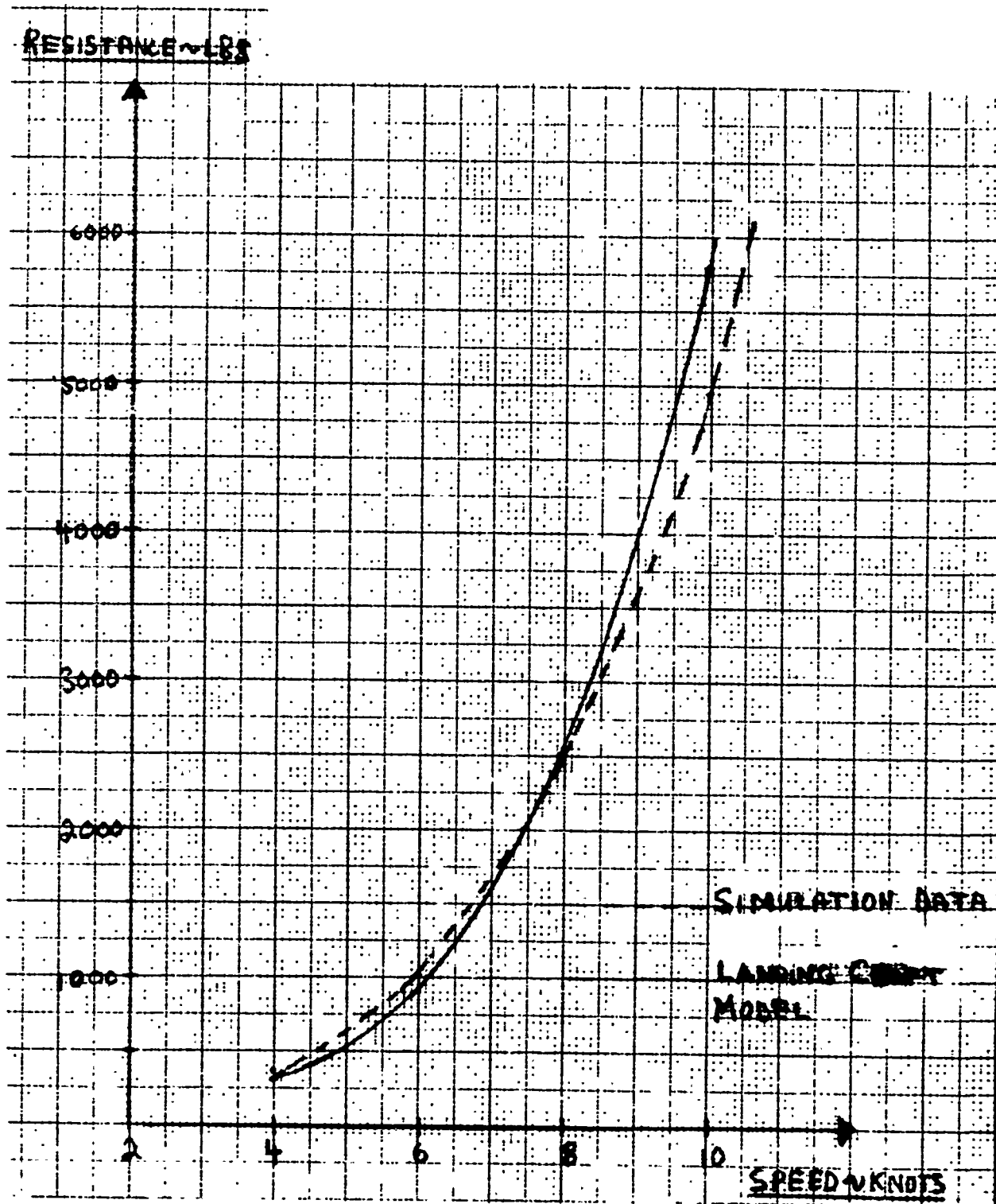


Figure 6. Variation of Resistance with Speed Ahead Twin Screws Calm Water

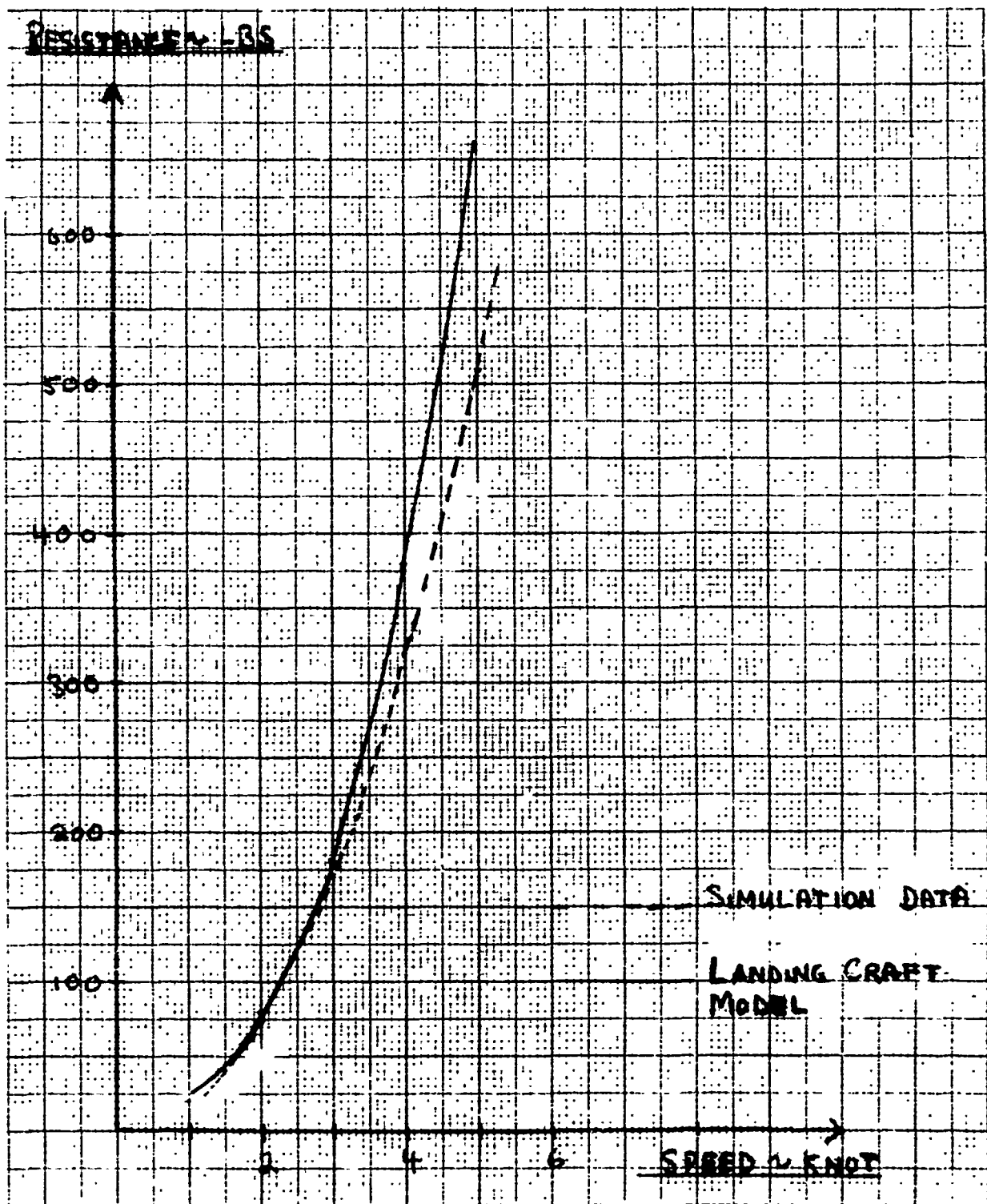


Figure 7. Variation of Resistance with Speed Astern Twin Screws Calm Water

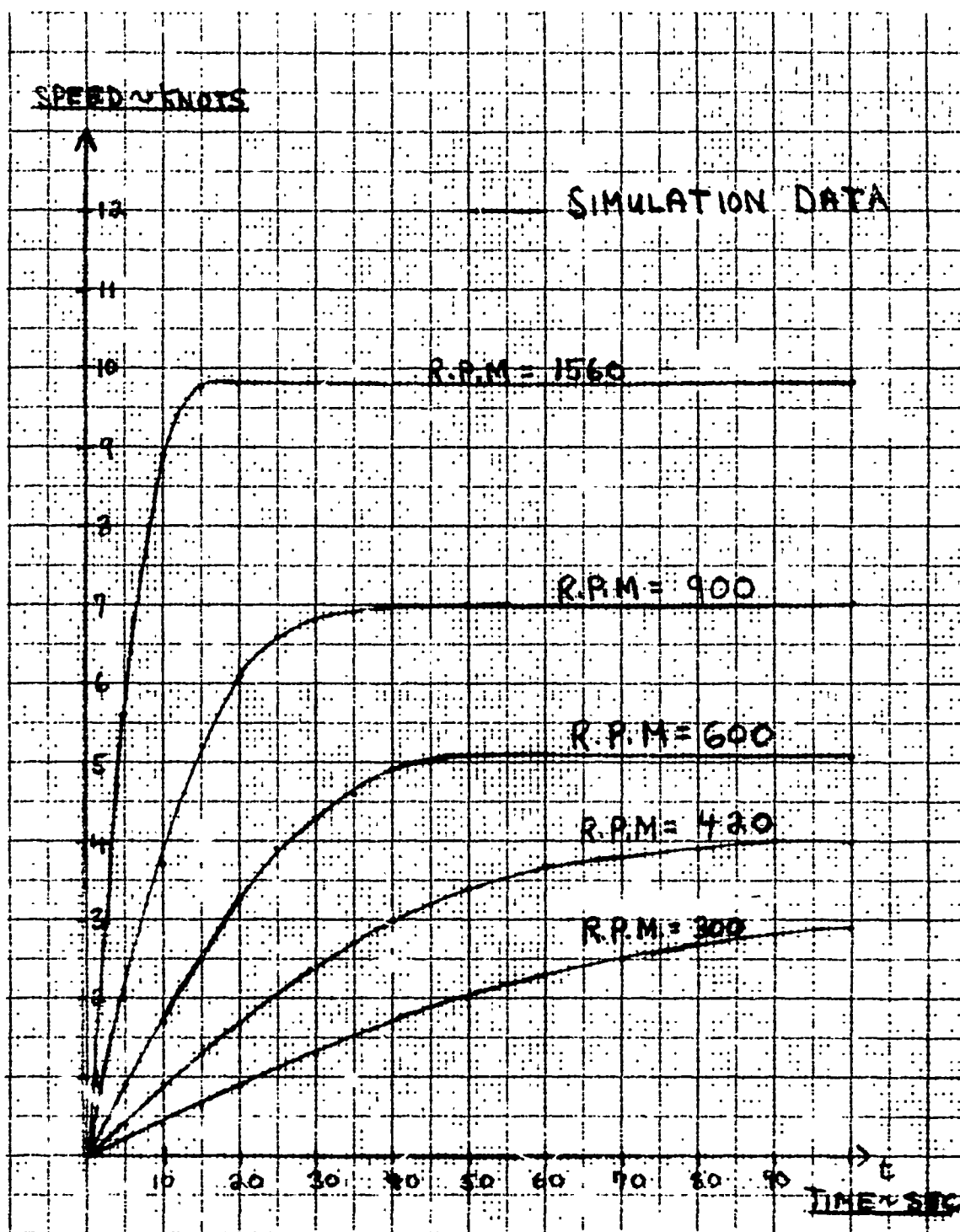


Figure 8. Computer Simulation of LCM Acceleration Curves Ahead Twin Screws Calm Water

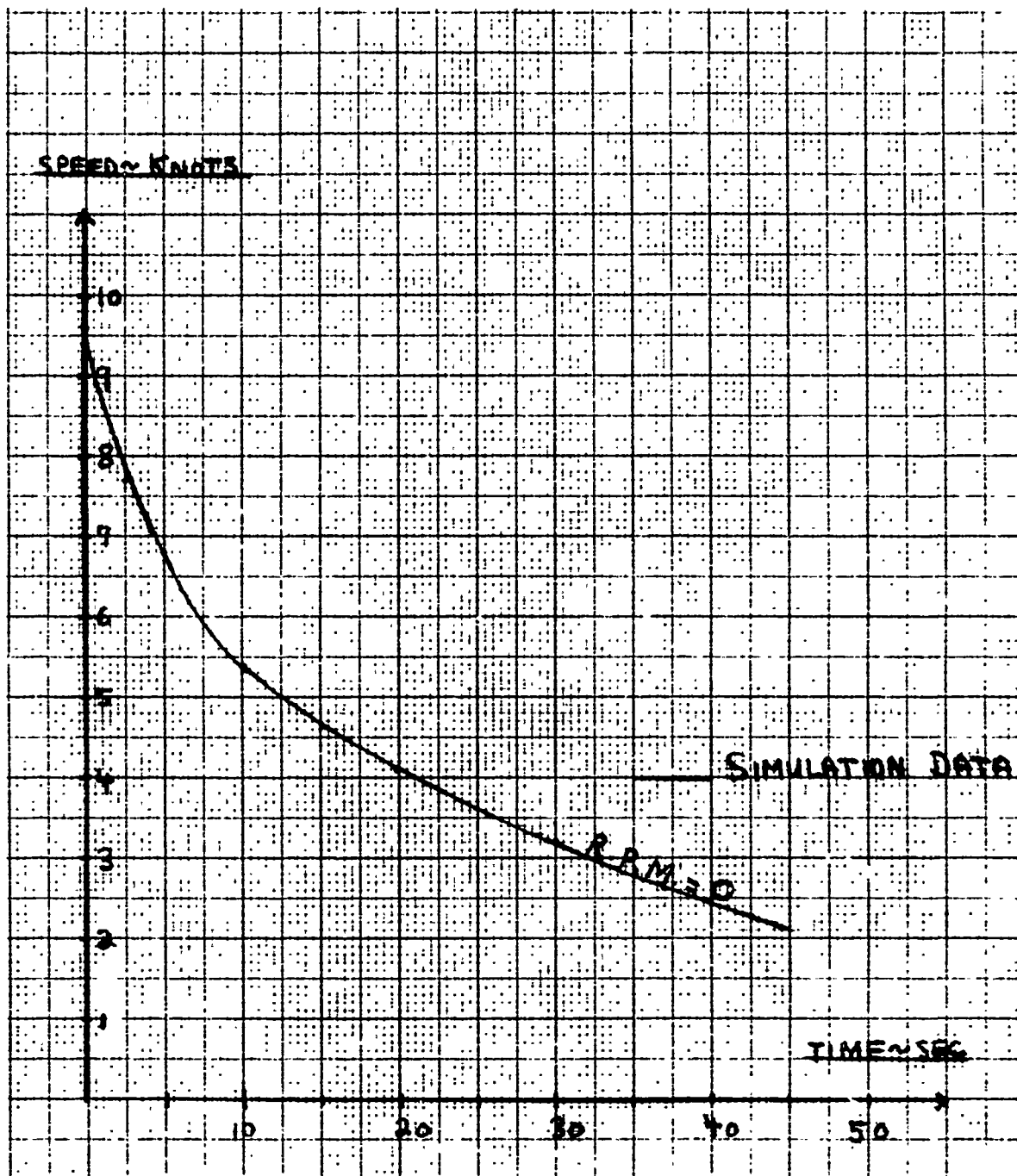


Figure 9. Computer Simulation of LCM Deceleration Curve When Engine Shut Off At Maximum Speed Ahead Calm Water

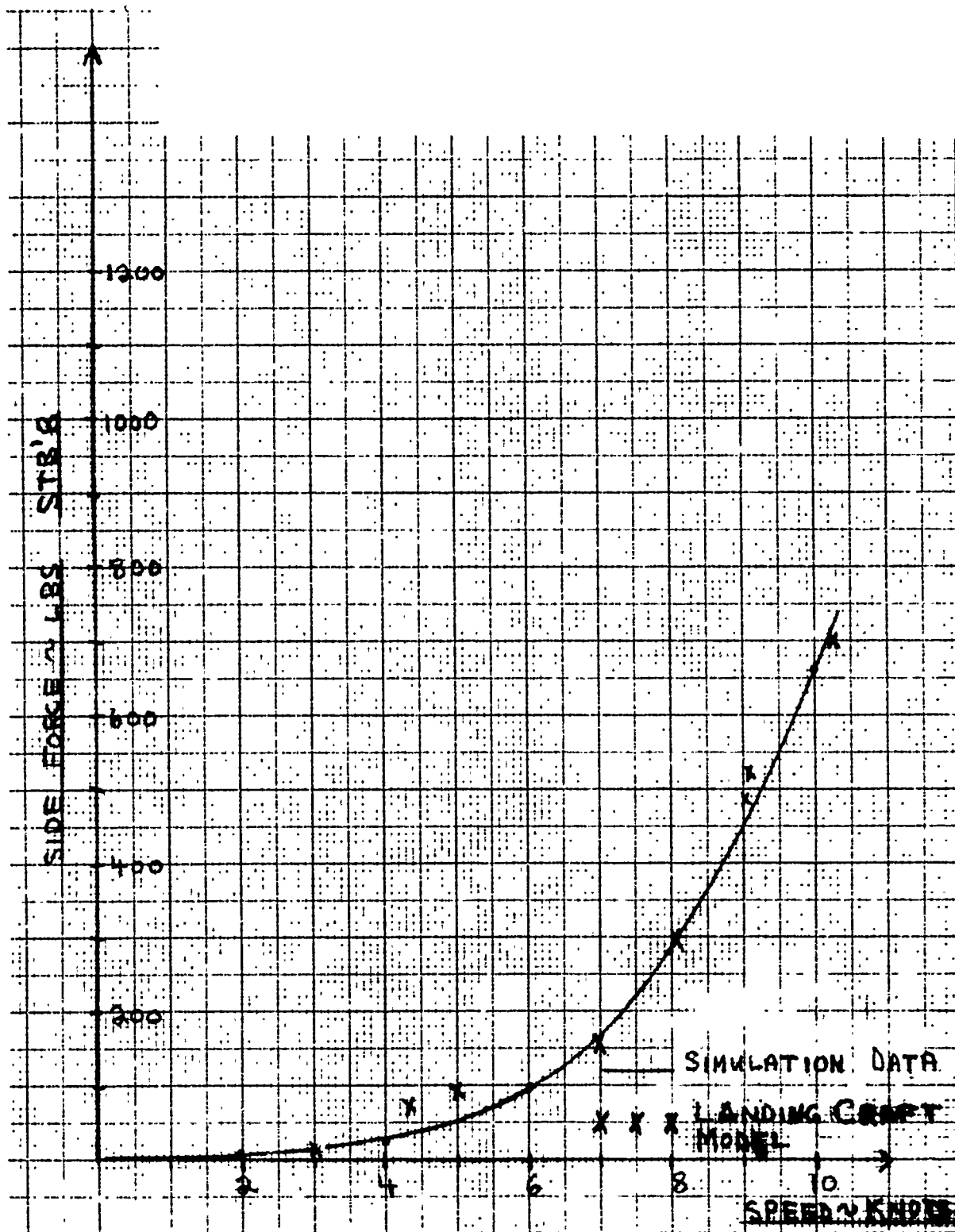


Figure 10. Variation of Side Force with Speed Ahead Twin Screws
Calm Water

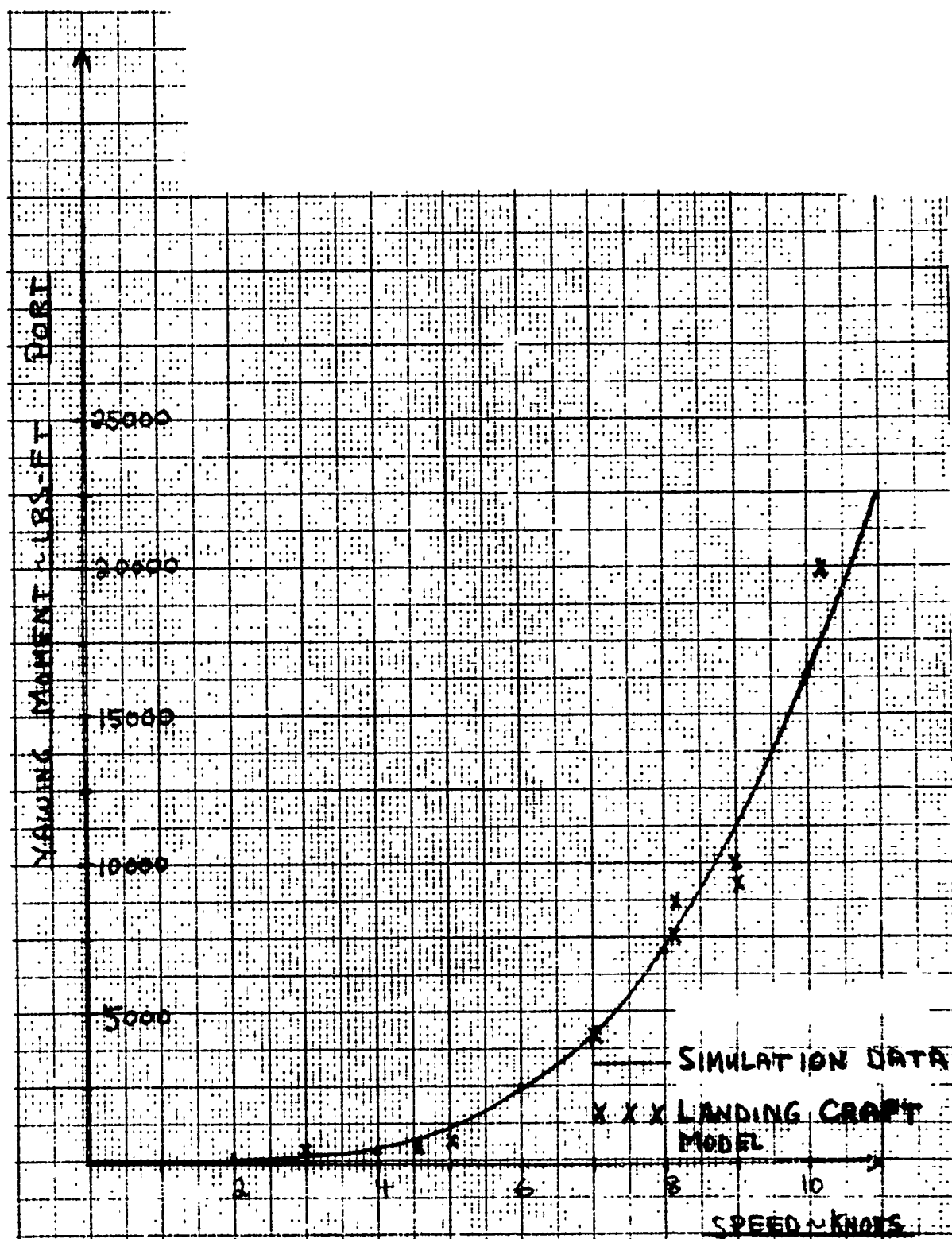


Figure 11. Variation of Yawing Moment with Speed Ahead Twin Screws
Calm Water

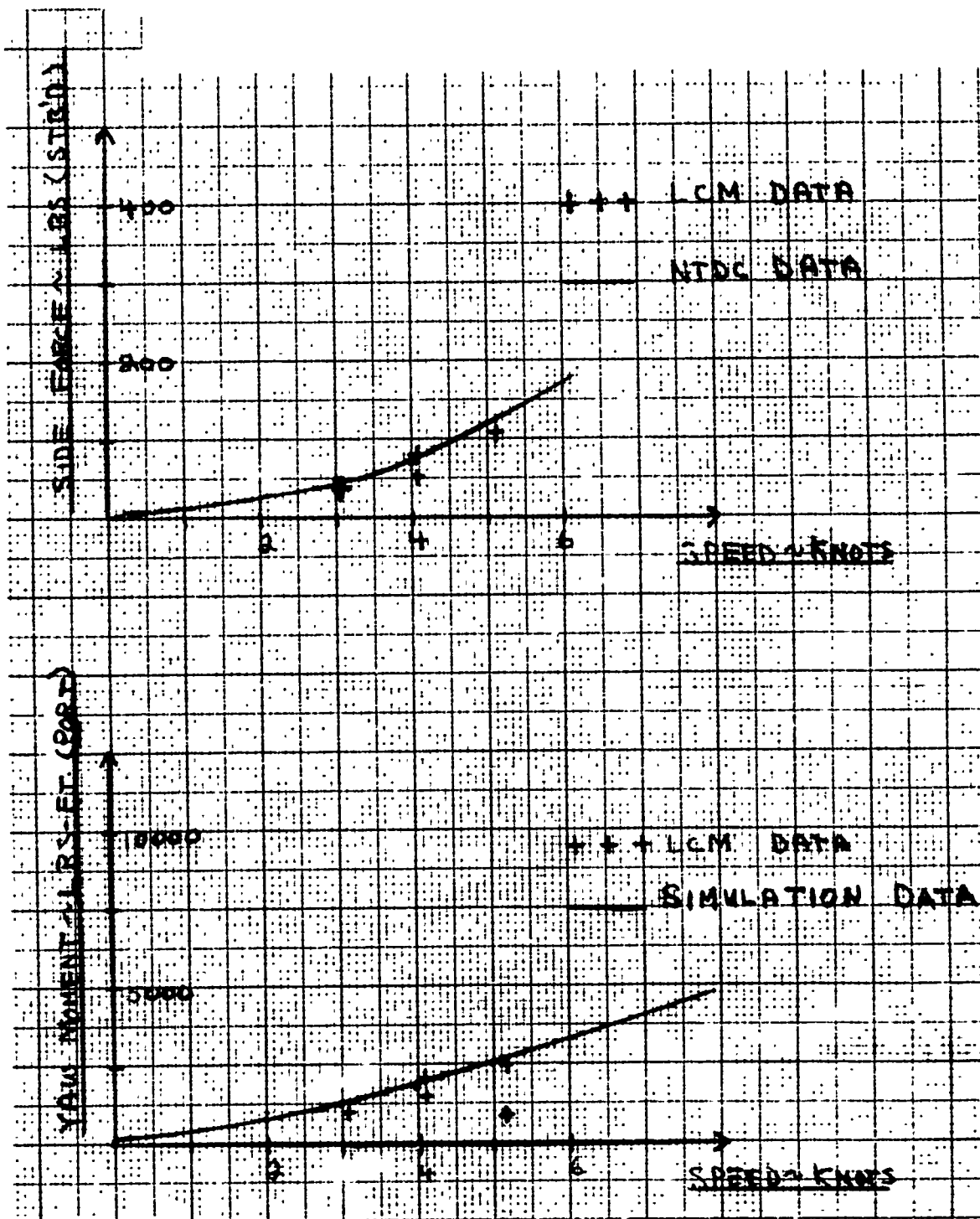


Figure 12. Variation of Side Force and Yawing Moment with Speed
 Astern Calm Water Twin Screws

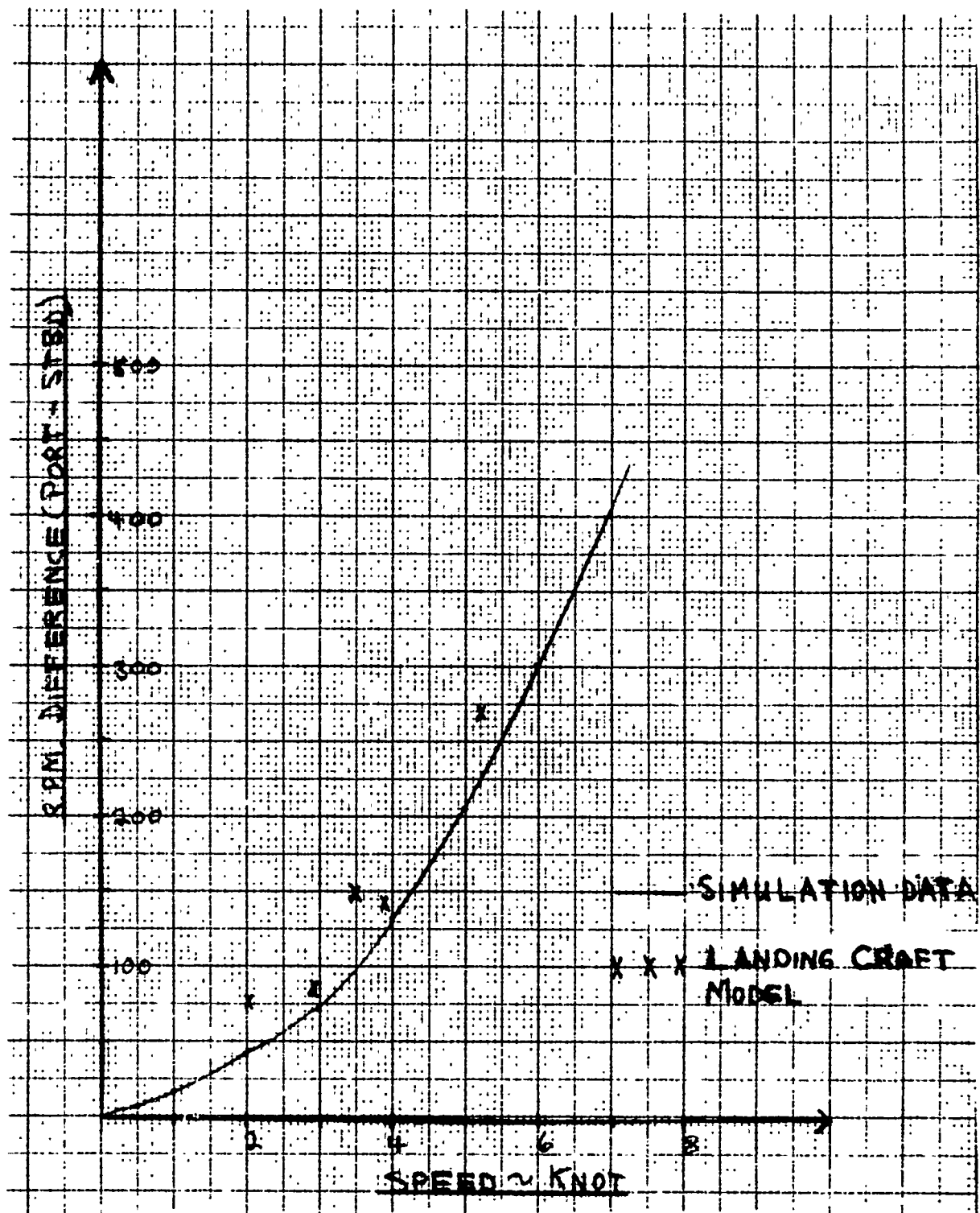


Figure 13. Variation of Difference of Props RPM with Craft Speed Ahead in Calm Deep Water

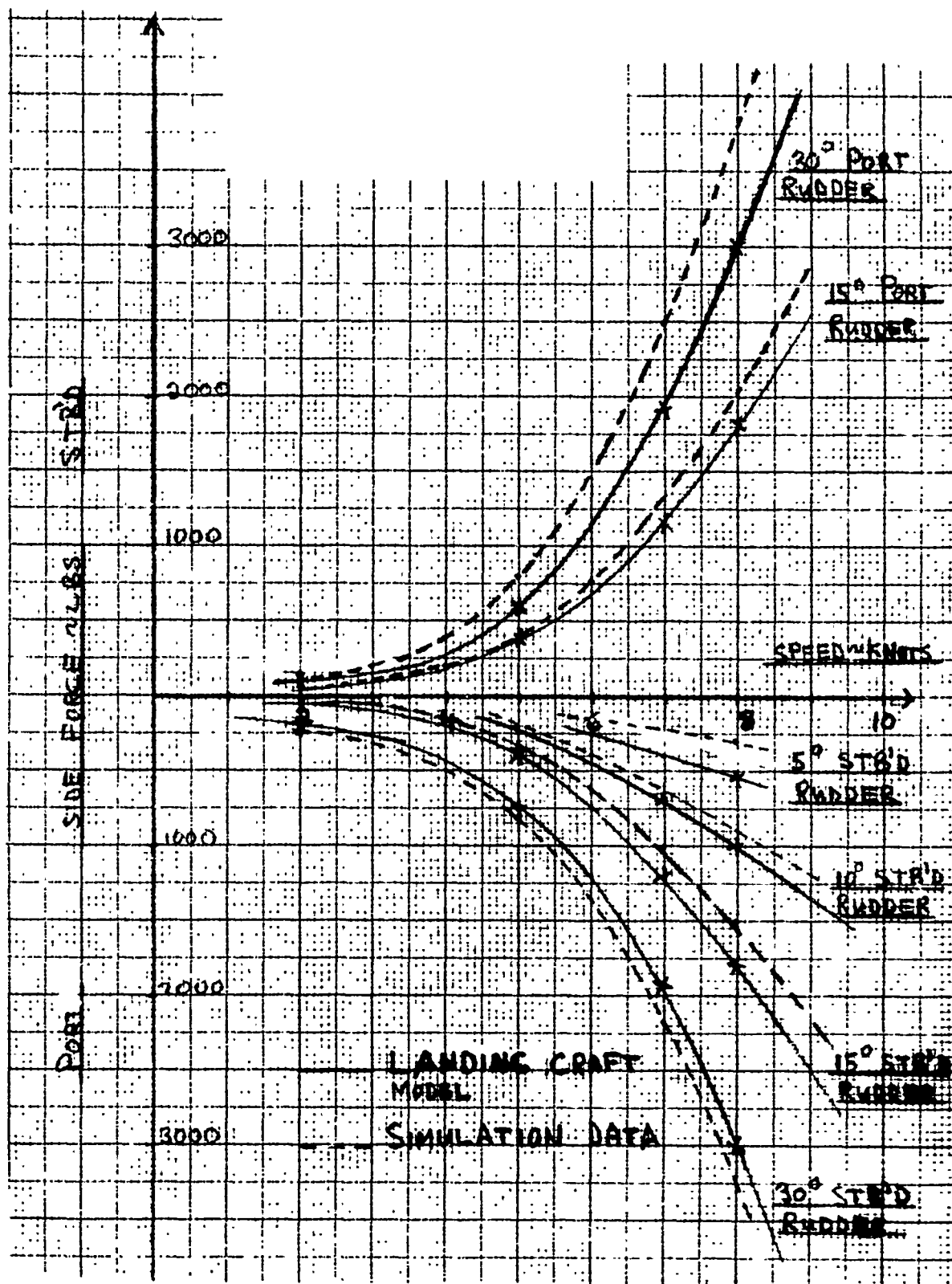


Figure 14. Variation of Side Force with Speed and Rudder Angle at Self Propulsion Ahead Twin Screws Calm Water

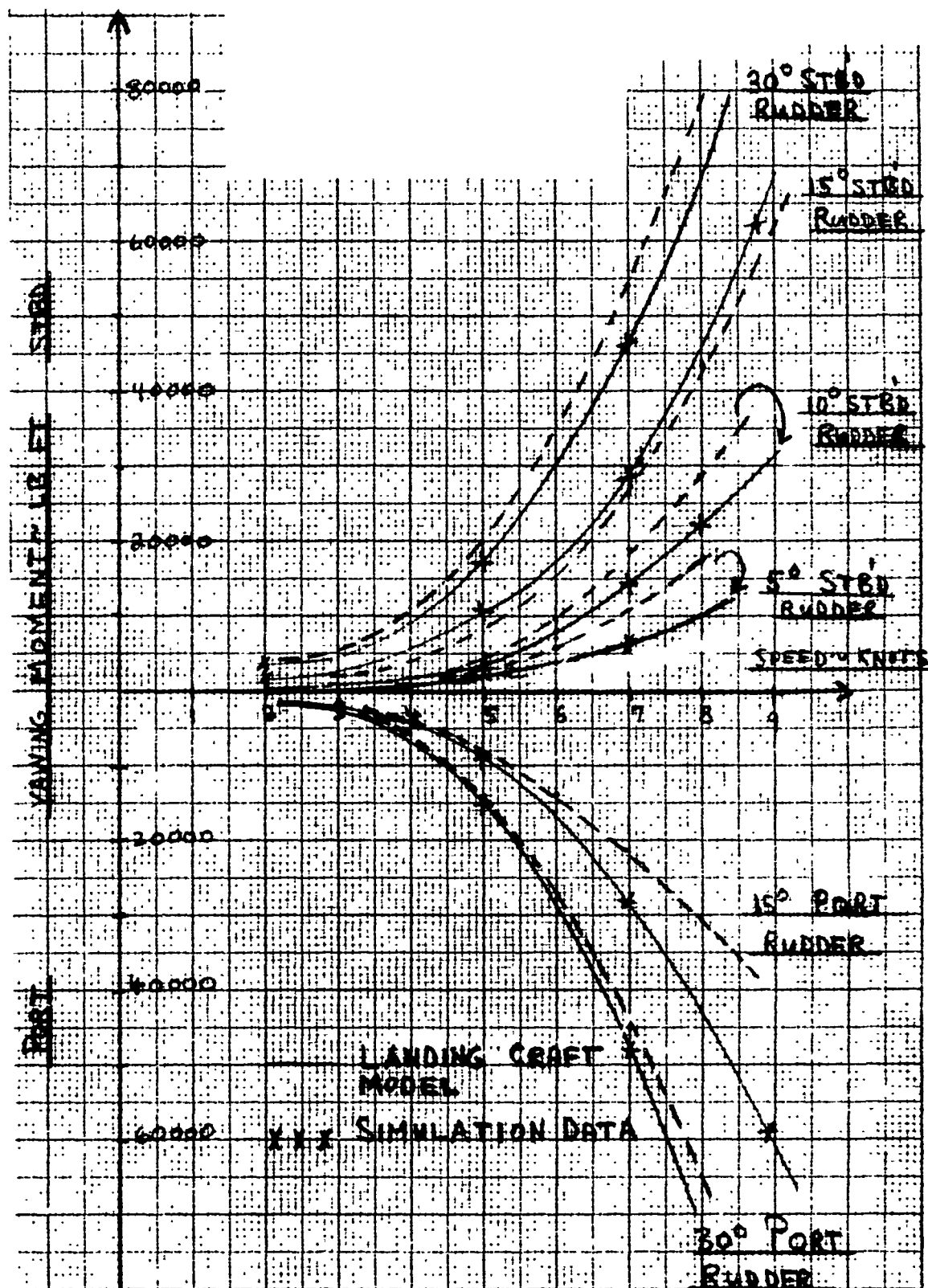


Figure 15. Variation of Yawing Moment with Speed and Rudder Angle at Self Propulsion Ahead Twin Screws Calm Water

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1. KAPLAN, PAUL; WARD, LAWRENCE W.; SARGENT, THEODORE P., A Mathematical Model for Assault Boat Motion in Waves, Technical Report No. NAVTRADEVCECEN 68-C-0152-1, June 1969, Oceanics, Incorporated, Plainview, New York.
2. KAPLAN, PAUL; WARD, LAWRENCE W.; CHUNG, YONG K., Final Report on Developments of a Mathematical Model Representing Assault Boat Motion in Waves, Technical Report No. NAVTRADEVCECEN 68-C-0152-2, July 1969, Oceanics, Incorporated, Plainview, New York.

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GLOSSARY

<u>SYMBOL</u>	<u>DESCRIPTION</u>
A_1	The effective frontal above-water area of the boats hull side.
A_2	The lateral above water area of the boat.
A'_{33}, N'_{zz}	Added mass and damping coefficient.
B	Beam width of boat.
C_D	Average drag coefficient of the boat.
g	Acceleration of gravity.
GM	Vertical distance between center of gravity and metacenter.
h	Water depth along sloping beach.
I_{xx}, I_{yy}, I_{zz}	Moment of inertial about the x, y, and z axes.
K, M, N	Moments (i.e., torques) about the x, y, and z axes. Also, referred to as the roll, pitch, and yaw moment.
K_p	A roll moment coefficient.
L	Boat length along load waterline.
$\frac{L}{2}$	Half-length of boat.
m	Mass of boat.
N_r, N_1, N_2, N_3	Hull force coefficients.
n	Propeller rotational speed, rev/sec.
n_o	Used to represent the effective rotational rate, rev/sec, of the port propeller.
n_p	Rotational rate, rev/sec, of the port propeller.
n_s	Rotational rate, rev/sec, of the starboard propeller.
p, q, r	Angular velocity components about the x, y, and z axes.
$T(n)$	Propeller thrust.
U_{RW}	Resultant wind velocity relative to the boat.
U_W	Resultant wind velocity relative to the inertial reference frame.

u, v, w	Velocity components in the direction of the x , y and z axes.
W	Weight of boat.
X, Y, Z	Components of force in the direction of the x , y , and z axes.
x, y, z	Right-handed orthogonal system of moving axes, fixed in the body; the z -axis is directed towards the bottom of the boat with the x - z axis the vertical plane of symmetry of the boat.
x_0, y_0, z_0	Right-handed orthogonal system of fixed axes, referred to an inertial reference frame fixed with respect to the surface of the earth. Used to describe the position of the boat relative to the shoreline.
X_u, Y_1, X_2, Y_3	Axial hull force coefficients.
Y_u, Y_1, Y_2, Y_3	Lateral hull force coefficients.
α	Beach slope.
δ	Rudder angle in radians about the z -axis. Positive if rotation is clockwise.
θ	Pitch angle, about the y -axis.
μ	Resultant wind vector angle with the negative x -axis.
ρ_a	Mass density of air.
ϕ	Roll angle, about the x -axis.
ψ	Yaw angle, about the x -axis.
ψ_w	Angle that wind vector makes with the inertial reference frame.